THEORETICAL CONSIDERATIONS ON VELOCITY CHANGE OF LAMB WAVES BY MAGNETOELASTIC EFFECT

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ABSTRACT

The propagation characteristics of the magnetoelastic Lamb waves in a YIG film magnetized tangentially are analyzed using perturbation technique. It is found that a velocity change by magnetic field is of order few percent and the waves attenuates near magnetic resonance frequency.

Introduction

Renewal interest in microwave acoustics in recent years has been directed towards controlling the characteristics of the SAW devices. Intrinsically, the propagation characteristics of the magnetoelastic(ME) waves are varied by adjusting external DC magnetic field. The possible ME surface wave applications like convolver, phase shifter and oscillator have been experimentally demonstrated [1 - 3]. The magnetic structure used in a experiment consists of an epitaxial YIG film with thickness of h, wherein the bias field H_i is tangential to the YIG film and parallel to the direction of wave propagation as shown in Fig.1. In the YIG film the ME Lamb waves can propagate rather than the ME Rayleigh waves discussed by Parekh [4].

The change of the propagation constant of the Lamb waves by ME effect is analyzed using pertubation technique.

The equatin of motion

The linear equations of motion of an exchange free ME medium which was described by Auld [5] can be rewrite into a simple form [6] as

$$\nabla_s \mathbf{v} = j\omega \mathbf{S} = j\omega(\hat{s}^H : \bar{T} + \hat{d}^{H*} \cdot \mathbf{h}) \ ,$$
$$\nabla \cdot \bar{T} = j\omega\rho\, \mathbf{v} \ , \tag{1}$$

and Maxwell's equatins with the quasistatic approximation can be expressed as

$$\nabla\cdot\mathbf{B} = \nabla\cdot(\hat{\mu}\cdot\mathbf{h} + \hat{d}^H : \bar{T}) = 0 \ , \ \mathbf{h} = -\nabla\phi \ . \tag{2}$$

In above equations v, T and h are the particle velocity stress and rf magnetic field, respectivily. The compliance matrix is given by

$$\hat{s}^H = \begin{bmatrix} S_{11} & S_{12} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{11} & S_{12} & 0 & 0 & 0 \\ S_{12} & S_{12} & S_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{S}_{44} & -jS_{45} & 0 \\ 0 & 0 & 0 & jS_{45} & \bar{S}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{44} \end{bmatrix} , \tag{3}$$

where

$$\bar{S}_{44} = \frac{1}{P}\left\{c_{44} - \frac{(b\gamma)^2\mu_0 H_i}{M[(\gamma\mu_0 H_i)^2-\omega^2]}\right\} ,$$

$$S_{45} = \frac{b^2\gamma\omega}{PM[(\gamma\mu_0 H_i)^2-\omega^2]} ,$$

$$P = \left[c_{44} - \frac{b^2\gamma}{M(\gamma\mu_0 H_i + \omega)}\right]\left[c_{44} - \frac{b^2\gamma}{M(\gamma\mu_0 H_i - \omega)}\right]$$

and the ME strain matrix is given by

$$\hat{d}^H = \begin{bmatrix} 0 & 0 & 0 & -jd_{14} & -d_{15} & 0 \\ 0 & 0 & 0 & -d_{15} & jd_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} , \tag{4}$$

where

$$d_{15} = \frac{\gamma^2\mu_0 b}{P[(\gamma\mu_0 H_i)^2-\omega^2]}\left(c_{44}\mu_0 H_i - \frac{b^2}{M}\right) ,$$

$$d_{14} = \frac{\mu_0\gamma\omega b c_{44}}{P[(\gamma\mu_0 H_i)^2-\omega^2]} .$$

In equations of (3) and (4), b, γ and M are the ME coupling coefficint, gyromagnetic ratio and saturation magnetization, respectivily, and also c is the stiffness. From these relations it is found that the ME medium is one which gives hybrid behavior of between the magnetostatic and elastic fields.

Perturbation theory

It is well known that the dispersion characteristics of the ME wave show the weak coupling between the magnetostatic and elastic wave near magnetic resonance frequency of f_h, ($f_h = \gamma\mu_0 H_{ih}/2\pi$) and elastic behavior far from resonance frequency [5]. Based on these facts our analysis is divided into two problems according to a frequency region considered.

i) Compliance matrix perturbation

In this section our analysis is limited to a frequency region far from magnetic resonance frequency and Maxwell's equations of (2) are neglected assuming that the rf magnetic field is nealy equal to zero. The source of perturbation is taken to be the compliance matrix of (3) which is slightly different from the ordinary compliance matrix of an isotropic elastic medium [7]. A compliance component of S_{45} within (3) means that the ordinary Lamb waves couple to the ordinary SH waves through the existence of ME coupling coefficint b.

Perturbation theory treated here is, in principle, similar to that used by Marcuse in formulating the coupling between optical transmission lines [8]. Assuming that the v and T are very nealy given by the superposition of the elastic fields of the SH and Lamb waves, we thus put

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$$\mathbb{v} = A\,\mathbb{v}_a + B\,\mathbb{v}_b + \nu,\quad \bar{T} = A\,\bar{T}_a + B\,\bar{T}_b + \boldsymbol{\tau}, \tag{5}$$

where A,B are the coefficint, ν , τ are small field disturbed by ME coupling.When the fields of v and T have the z dependence of the form $\exp(-j\beta_i z)$,the unperturbed elastic field of either wave satisfy the equations from (1)

$$\begin{aligned} &\nabla_{st}\,\mathbb{v}_i - j\beta_i\,\hat{z}\cdot\mathbb{v}_i = j\omega\hat{s}_i : \bar{T}_i \quad, \\ &\nabla_t\cdot\bar{T}_i - j\beta_i\,\hat{z}:\bar{T}_i = j\omega\rho\mathbb{v}_i \quad, \\ &(i = a,b\ ). \end{aligned} \tag{6}$$

The $\hat{S}_i$ is the compliance matrix to contribute the waves i in the absence of the ME coupling.

Substituting (5) into (1) and after cancelations of a number of term with the help of (6),we obtain the simple relation as a function of v_i and T_i.By applying the reciprocity relation of the elastic field to the equations obtained and integrating over cross sectional area of the wave guide shown in Fig.1,we can derive the final relation which determines the change of the propagation constant by ME effect assuming that nondependence of the fields in the y direction as

$$\beta = \frac{(\beta_a + \beta_b) + \frac{I_{aa}}{4P_{az}} + \frac{I_{bb}}{4P_{bz}} \pm \sqrt{D}}{2}, \tag{7}$$

where

$$D = \left(\beta_a - \beta_b + \frac{I_{aa}}{4P_{az}} - \frac{I_{bb}}{4P_{bz}}\right)^2 + \frac{4I_{ab}I_{ba}}{4P_{az}4P_{bz}} \quad,$$

$$I_{aa} = \omega\Delta S_{44}\int_0^h |T_{5a}|^2 dx \;,\quad I_{bb} = \omega\Delta S_{44}\int_0^h |T_{4b}|^2 dx \;,$$

$$I_{ab}I_{ba} = \omega^2 S_{45}^2 \int_0^h T_{4b}^* T_{5a}\,dx \int_0^h T_{5a}^* T_{4b}\,dx \;,$$

$$\Delta S_{44} = \bar{S}_{44} - S_{44} \;,$$

and $4P_{az}$,$4P_{bz}$ are the average power flow of the Lamb and SH waves,respectivily.

The typical magnetic field dependence of the propagation constant is presented in Fig.2 for two different film thicknesses assuming that the lowest antisymmetrical mode of the Lamb waves couples to the lowest mode of the SH waves.For the evaluation of (7), numerical values for frequency is chosen to be 294 MHz. In Fig.2 $(\beta-\beta_a)/\beta_a = \Delta\beta_a/\beta_a (\Delta V_a/V_a)$ applies to the propagation constant which is deviated from that of the ordinary Lamb wave while $(\beta-\beta_b)/\beta_b = \Delta\beta_b/\beta_b$ applies to the SH wave.The frequency dependence of the propagation constant,that is ,dispersion diagram is presented in Fig.3 for three different thickness resonance modes of the SH wave ($n\pi/h$,n=0,1,2),wherein the magnetic field intensity of 100 Gauss is fixed.It can be seen in figures that a sign of $\Delta\beta/\beta\ (\Delta V/V)$ changes as the magnetic field increases through the magnetic resonance point and that the variation of the propagation constant (velocity) is of order few persent except for a frequency lying in the magnetic resonance.

ii) Perturbation by the magnetostatic waves

In this section our analysis is limited to a frequency region near magnetic resonance.The source of perturbation is taken to be a Maxwell's equations,that is,the magnetostatic field which is derived from (2) [9]. The change of the propagation constant by a weak coupling between the magnetostatic and elastic waves through ME coupling coefficint b is caluculated.We repeat here the same analytical procedure as that of section (i) putting the fields as

$$\bar{T} = A\,\bar{T}_a + \bar{\tau}\;,\quad \mathbb{v} = A\,\mathbb{v}_a + \nu,\quad \mathbb{h} = B\,\mathbb{h}_b + \vartheta, \tag{8}$$

where v_a ,T_a and h_b are the unperturbed fields and correspond to the fields of the ordinary Lamb wave and the magnetostatic volume wave,respectivily. τ , ν and ϑ are the small fields disturbed by a coupling of these waves.

After some mathematical manupulations using (1),(2) and (8),we can obtained the relation which determines the change of the propagation constant by ME effect.

Since the dispersion diagram of the magnetostatic volume wave with propagating the direction of the magnetic field shows a negative group velocity characteristic [9] ,active coupling occurs at crossover point between the dispersion curve of the ordinary Lamb and magnetostatic waves.As the results the propagation constant of the ME Lamb wave becomes complex $\beta = \beta_{real} + j\Delta\alpha$ showing the attenuation of the wave,where $\Delta\alpha$ is

$$\Delta\alpha = j\omega\left[\frac{\int_0^h T_{5a}^* d_{15} h_{xb}\,dx \int_0^h d_{15} T_{5a} h_{xb}^*\,dx}{4P_{az}4P_h}\right]^{\frac{1}{2}}, \tag{9}$$

and $4P_h$ is the total power flow of the magnetostatic wave.

The typical dispersion diagram,which also defines the change of the propagation constant by the magnetostatic wave,is presented in Fig.4 for 10 μ film thickness and magnetic field intensity of 100 Gauss assuming that the lowest antisymmetric mode of the Lamb wave couples to the lowest mode of the magnetostatic volume wave.From the numerical estimations of (9) for different film thicknesses,it is found that the attenuation constant of $\Delta\alpha$ increases with decreasing the film thickness.

Conclusion

The propagation characteristics of the ME Lamb wave have been trated using perturbation theory It is shown that the velocity change of order few percent is accomplished by the magnetic field and frequency and also that near magnetic resonance frequency the amplitude of the ME Lamb wave decreases as the wave propagates.

The change of the propagation constant obtained here is slightly larger value compared with the experimental results by Volluet [3] .This is due to the mass loading effect of the gadlinium gallium garnet substrate which is neglected in our analysis

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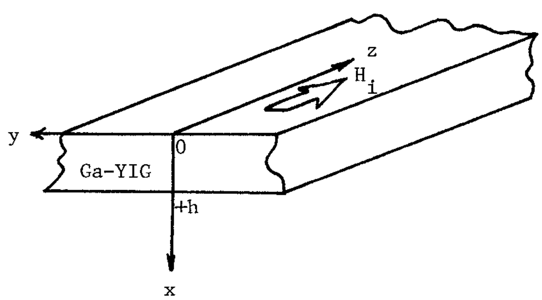

Fig.1 Schmatic illustration of YIG film geometry.

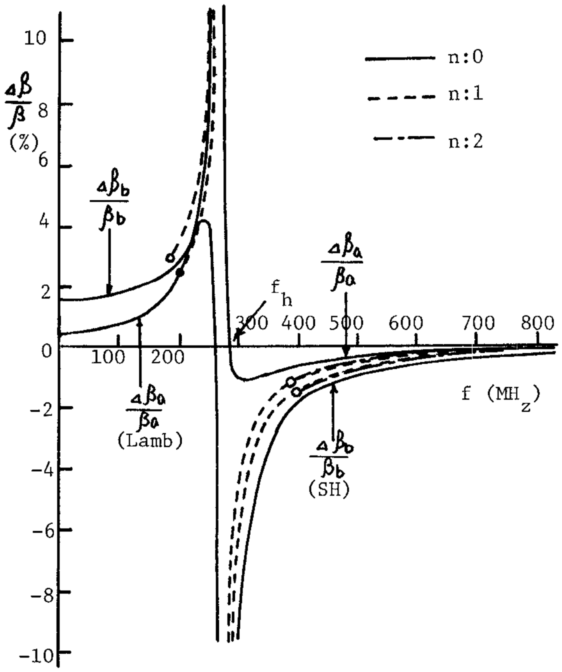

Fig.3 Computed dispersion diagram of Lamb waves coupled with different thickness resonance modes of the SH wave.The fixed parameters are $\mu_0 H_i$:100 Gauss, $\mu_0 M$:300 Gauss and h:10μ .

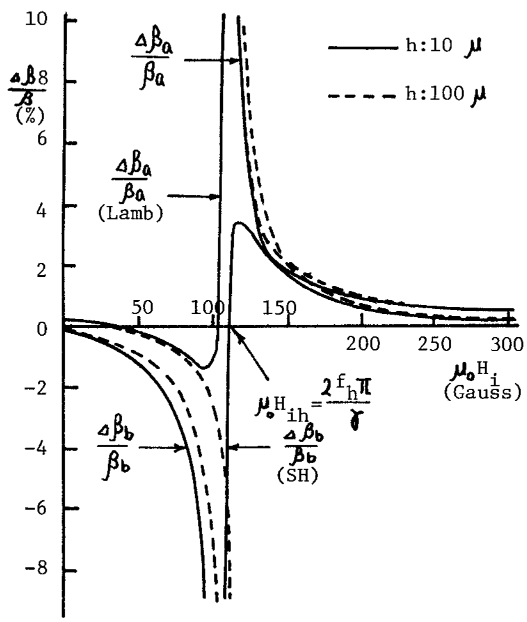

Fig.2 The change of the propagation constant by the magnetic field for different film thicknesses.The fixed parameters are frequency:294 MH_z,n:0 and $\mu_0 M$:300 Gauss.

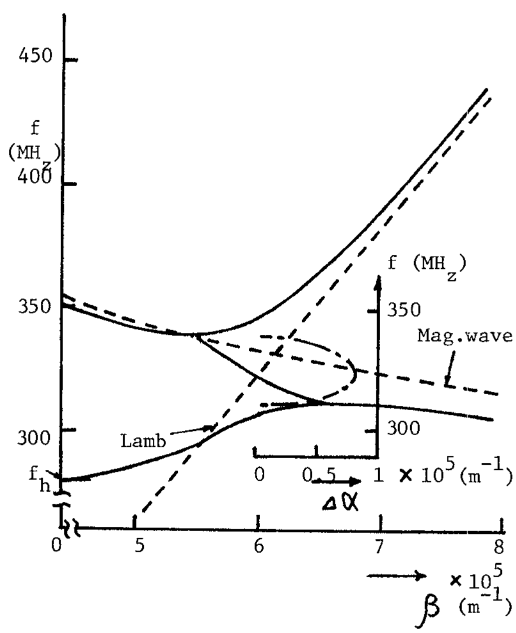

Fig.4 Computed dispersion diagram of ME Lamb waves near magnetic resonance frequency. ——— :perturbed solution,------ :unperturbed solution and —·—·— :attenuation constant.